

## Linear Programming Model and Sensitivity Analysis of Selected Agrarian Constraints Faced by Farming in The Province of Khyber Pukhtunkhwa

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### ABSTRACT

*Being an agrarian economy, the role of agriculture in the economic structure of Pakistan cannot be ignored. Inadequate agrarian structure and a number of constraints adversely affect the productivity. This article focuses on very few of the main constraints, which restrict the farm efficiency. This article addressed the constraints which have binding and limiting effects on the farm efficiency. It also identified the sensitivity of changes in certain inputs on the productivity. For the achievement of the aim of the research article, the most irrigated area of the province that is the Peshawar valley was included in the target area of the study. Using the stratified random sampling technique ten villages from the three randomly selected districts of Peshawar valley were chosen purely on random basis. It was proposed that a sample size of 150 farmers would be the true representative of the whole universe. The villages and the sample farmers were selected, applying the simple random sampling technique. The Linear Programming model and Sensitivity Analysis were used as analytical techniques.*

**Key Words:** Linear Programming, Sensitivity Analysis, Constraints, Activities, Shadow Prices and Economic Efficiency

### INTRODUCTION

Being an agrarian economy, the role of agriculture in the economic structure of Pakistan cannot be ignored. This sector in fact involves indirectly the illiterate and unskilled working population of the rural areas in the development process of industrialization. The agrarian Structure of an economy plays a key role in enhancing agricultural productivity and raising economic efficiency of farms. The present study focuses on describing in detail the existing agrarian structure in Khyber Pukhtunkhwa and determining the economic efficiency of different farms, applying the Linear Programming and Sensitivity analysis.

The analysis carried out in various researches of agricultural sector in the province of Khyber Pukhtunkhwa reveal that inadequate agrarian structure, that is, the very smallness of the small farms and the very largeness of the large farms, tenurial status, irrigation status and mechanization state, are among the main constraints, which have adversely affected the farm efficiency. Lack of knowledge about the recommended farm practices also has a direct bearing on the efficient utilization of resources.

It could be argued that it is not enough to know about the constraints only. It would be useful to know which of the constraints have had binding and limiting effects on the farm efficiency. Linear Programming (LP) technique helps in by sorting out the effects of such constraints on economic efficiency of the farms.

Linear Programming is a method of identifying constraints, both of a technical (e.g. water) or of an institutional (e.g. credit) nature, which have a significant effect on a multi-product

or multi- activity farming system. The sensitivity analysis further enables to analyze the effects of any change in the constraints on the overall farm activity.

According to Jhol & Kapur (1967) Linear programming is a more systematic and accurate method of determining mathematically the optimum combination of inputs so as to maximize the income or minimize the cost within the limits of available resources. Linear programming model is the most important tool in economic theory for optimizing production and minimizing cost. For its mathematical accuracy and scientific standard calculation, particularly in the limited farm resource condition, linear programming technique is highly useful and can be used in any economic situation where there exists a framework of (i) resource restrictions, (ii) alternative activities or processes and (iii) quantifiable objective viz, maximization of profit or minimization of cost.

LP is equally capable of accommodating a large quantum number of variables and is simple enough in terms of data requirements. Therefore, in the environment of Pakistan and especially in the province of Khyber Pakhtunkhwa, the Linear Programming can prove to be a suitable technique used in this study. The sensitivity of variations in main inputs can be analyzed using the Sensitivity Analysis.

### OBJECTIVES

The main objectives of the articles are:

- To identify the constraints which have binding and limiting effects on the efficiency of farms and
- To assess the sensitivity of effect of any change in the constraint(s) on the farm activity.

### RESEARCH METHODOLOGY

The issues mentioned in the objectives of the study indicate that the study is primarily based on the primary data though in certain cases data was secured from the secondary source. For the collection of primary data a comprehensive Interview Schedule was evolved.

#### Sampling Design and Sample size:

A number of constraints can be dealt with using Linear Programming Model. For the purpose of this study certain major constraints have been considered. One of them was the availability of irrigation water from the canal source. This situation could bitterly be analyzed in the area of the province, where the canal source of irrigation is in abundant. On the basis of past researches in the relevant field it was observed that Peshawar Valley, comprising of well known irrigated districts like Charsada, Mardan, Swabi, Nowshera and Peshawar was very rich in this regard. Therefore this could be the target area of the study. It was assumed that from the irrigation and some other basic constraints point of view, homogeneity existed in these districts. Therefore, the Multi- Stage/Stratified Random Sampling technique was used. At the first stage three districts called strata were chosen on random basis. At the second stage, the number of sample villages called sub-strata from each selected district were randomly selected, which were proportionate to the number of villages in each district. At the third stage, the sample farms were selected using the simple random sampling technique.

Homogeneity was assumed in the basic characteristics of the villages drawn from the selected districts, hence even a small number of villages, selected on random basis would fairly represent the whole stratum. It was proposed that ten sample villages would represent the whole target area. Similarly homogeneity was assumed in the characteristics of the farms located in the same sub-strata, therefore, a sample size of 150 farms/farmers would be the true representative of the whole, if it was drawn randomly. A list of all the farmers was prepared for the selected villages from the selected districts. This list served as a sampling frame for this study.

For proportionate distribution, the following expression was applied:

$$n_s = \frac{nN_s}{N}$$

Where

$n_s$  = Sub-sample drawn from the sth stratum  
 $n$  = Total size of the sample (i-e 150)  
 $N_s$  = Sub-Population of the sth stratum  
 $N$  = Total Population of the target area

#### Formulation of Linear Programming (LP) model:

The following Linear Programming Model was developed for this article to explore the possibilities of optimizing the returns by considering various activities that is crop production under different tenure statuses.

The objective function was to maximize the Farm Profit(Net Revenue) that is:

$$\text{Max. profit} = \text{Max. } Y = \sum_{i=1}^n P_i X_i$$

Subject to the constraints:

$$1) \quad \text{Land} = \sum_{i=1}^n \sum_{j=1}^{12} AX_{ij} \leq A$$

$$2) \quad \text{Water} = \sum_{i=1}^n \sum_{j=1}^{12} WX_{ij} \leq W$$

$$3) \quad \text{Labour} = \sum_{i=1}^n \sum_{j=1}^{12} LX_{ij} \leq L$$

$$4) \quad \text{Bullock hours} = \sum_{i=1}^n \sum_{j=1}^{12} BX_{ij} \leq B$$

$$5) \quad \text{Working capital} = \sum_{i=1}^n \sum_{j=1}^{12} KX_{ij} \leq K$$

The non-negativity constraints were  $X_i \geq 0$

Where

Y	=	Profit (Net Revenue, in Rs.)
$P_i$	=	Profit (Net Revenue) per acre from crop/activity 'i' (in Rs.)
$X_i$	=	Area under crop 'i' (in acres)
$AX_{ij}$	=	Area under crop 'i' in the jth quarter (in acres).
$WX_{ij}$	=	Water used for crop 'i' in the jth quarter (in inches/acre),
$LX_{ij}$	=	Human Labour used for crop 'i' in the jth quarter (in hours).
$BX_{ij}$	=	Bullock Labour used for crop 'i' in the jth quarter (in hours).
$KX_{ij}$	=	Working Capital allocated to crop 'i' in the jth quarter (in Rs.)
A	=	Total availability of Land (in acres)
W	=	Total availability of Irrigation Water (in inches/acre)
L	=	Total availability of Human Labour (in hours)
B	=	Total availability of Bullock Labour (in hours)
K	=	Total availability of Working Capital (in Rs.)
i	=	Denoting crops, which vary from 1,2,3, ... n ( 12 Major Crops)
j	=	Denoting months of the year, which vary from 1,2,3, ... 12.

#### EXPLANATION OF THE MODEL

**Objective Function:** The objective function of the model was to maximize the profit 'Y' ( Net Revenue ) of the sample farms, subject to the technical constraints of the production function and the level of resource availability.

**Selected Activities:** The cultivation of a crop was termed as an activity. For the purpose of this LP model, the following crops from irrigated areas were considered as “activities”:

- |                         |                      |
|-------------------------|----------------------|
| 1. Wheat                | 2. Barley            |
| 3. Maize                | 4. Sugarcane         |
| 5. Rice                 | 6. Rabi Pulses       |
| 7. Kharif Pulses        | 8. Rabi vegetables.  |
| 9. Kharif vegetables    | 10. Rabi fodder      |
| 11. Kharif fodder       | 12. Other Rabi crops |
| 13. Other Kharif crops. |                      |

#### Technical Constraints:

The Technical Constraints were divided into two categories, namely:

- Fixed Resource Constraints (including land, canal water; family labour and bullock labour).
- Special Constraints that restricted the range of feasible cropping patterns.

Due to the seasonal nature of crop production, the total amount of various resources and their availability during different stages of production was important. For this purpose, the amount of land, canal water, family labour hours, and bullock hours were expressed each as 12 monthly constraints. The analysis of various individual constraints was as follows:

**a) Fixed Resource Constraints:** The main fixed resources were discussed below:

**i) Land Constraint:** The acreage available for growing crops was determined by the farm size. A 7.5-acre farm was taken as the representative sample size, corresponding to the average size of the farms emerging from the field survey. The land constraint was expressed as 12 monthly constraints.

**ii) Family labour Hours Constraint:** Again, based on the field study survey, the average family labour hours available on a 7.5 acre farm were taken as 250 man hours per month. The family labour constraint was expressed as 12 monthly constraint.

**iii) Bullock Hours Constraint:** The field survey showed that on average, one pair of bullocks was kept on a 7.5 acre farm. It was assumed that the bullocks could actively work for 25 days in a month and 8 hours per day, which gave 200 bullock hours per month on a 7.5 acre farm. The bullock hours constraint was also expressed as 12 monthly constraint.

**iv) Canal Water Constraint:** Without a tube well and other sources of water availability was limited by the canal water, which was supplied by the government, and this amount varied with the size of the farm. The data on the monthly acre inches of canal water available at farm level were derived from the field survey and were enumerated along with other fixed resources, available to 7.5 acre farm in following table 1.

Table 1. The fixed Resources Available to a 7.5 acre Farm

Month	Fixed Resources			
	Land (acres)	Family Labour (in man hours)	Bullock Labour (in bullock hours)	Canal water (acre Inches)
April	7.5	11.85	200	312
May	7.5	17.12	200	312
June	7.5	19.93	200	312
July	7.5	20.00	200	312
August	7.5	20.14	200	312
September	7.5	18.50	200	312
October	7.5	17.10	200	312
November	7.5	12.76	200	312
December	7.5	12.94	200	312
January	7.5	04.99	200	312
February	7.5	09.76	200	312
March	7.5	09.15	200	312

Source: Field Survey

**b) Special Constraints:** There were some other constraints in addition to the above mentioned fixed resource constraint, which restricted the feasibility of cropping activity of the representative farm acreage (7.5 acres). These constraints included minimum acreage of fodder for bullocks and acreage of wheat/maize for domestic consumption. The working capital constraint also limited the crop production. This was assumed equal to the total savings of the farm households.

**SOLUTION OF THE LP PROBLEM**

For the solution of this LP problem the simple method has been used. The computations have been carried out by the computer. It is assumed that the availability of working capital is Rs. 3,000 per year with additional tubewell water to the extent of 25 percent of the total

irrigation water. The activity wise results of the model by tenurial status are presented in the following table 2, in addition, the cropping pattern, the total cropped area, cropping intensity and net revenue are also indicated in the table2 below:

Table2. Activity levels of the basic model

Description	Tenurial status		
	Owner	Owner-cum-tenant	Tenant
<b>i) Crops:</b>			
<b>a) Rabi Crops:</b>			
Wheat	1:45	1.40	1.55
Barley	1.25	1.15	--
Pulses	0.95	0.80	0.25
Vegetable	115	0.95	1.85
Fodder	0.35	0.65	1.10
Other crops	0.20	0.25	0.35
<b>Subtotal (a)</b>	<b>5.35</b>	<b>5.20</b>	<b>5.15</b>
<b>b) Kharif Crops:</b>			
Maize	1.30	1.35	1.40
Sugarcane.	1.10	1.00	0.90
Rice	0.85	0.85	0.70
Pulses	0.55	0.40	--
Vegetable	0.95	1.00	0.90
Fodder	0.25	0.30	0.50
Other crops	0.15	0.20	0.55
<b>Subtotal (b)</b>	<b>5.15</b>	<b>5.10</b>	<b>4.95</b>
<b>(ii) Total Cropped Area (A+B)</b>	<b>10.50</b>	<b>10.30</b>	<b>10.10</b>
<b>(iii) Cropping Intensity</b>	<b>140.00%</b>	<b>137.33%</b>	<b>134.67%</b>
<b>(iv) Shadow Prices of Resources (Rs.)</b>			
<b>a) Water (Rs.)</b>			
Water (February)	90.75	89.85	88.00
Water (April)	270.20	265.50	262.50
Water (July)	75.25	75.00	70.25
Water (November)	185.16	180.25	175.50
<b>b) Working Capital (Rs.)</b>	<b>2.31</b>	<b>2.52</b>	<b>2.95</b>
Net Revenue (Profit):			
Annual Total (Rs.)	8541.77	7856.16	7309.13

### ANALYSIS OF THE LP PROBLEM

According to the results of the model, the profit maximizing owners, owner-cum-tenants and the tenants should grow 10.50, 10.30 and 10.10 acres of land respectively. For reaching the optimal allocation of land and water, the "shadow prices" of the working capital and the water constraints have indicated very high values.

It is observed that the cropping intensities obtained from the basic solution of LP model have been 140.00%, 137.33% and 134.67% on the farms operated by owners, owner-cum-tenants and tenants respectively. The different types of farms are in conformity with the differences in the allocation of fixed resources among crops.

The high shadow prices of water in a particular month of a year indicates its shortage in that which is confirmed by the results of LP model, indicating a significant shortage of water in the month of April followed by the month of November. The shadow prices of water in the months of April and November have been estimated at Rs. 270.20 and Rs. 185.16 respectively in the optimum farm plan of the owner farmers, this implies that the supply of an additional acre inch of water has increased the net revenue by Rs. 270.20 and Rs. 185.16 respectively. Similarly the net revenue of the 'owner-cum-tenants and tenants can be enhanced if the supply of an additional acre inch of water is increased. It can be concluded from the above analysis that water supply in the month of April and November has remained an effective constraint on the economic efficiency of the farms.

The shadow prices of the working capital have been estimated at Rs. 2.31, Rs. 2.52 and Rs. 2.95 on farms of owners, owner-cum-tenants and tenants respectively, showing that there has been the shortage of capital on these farms. This equally implies that an increase of one rupee in the working capital would rise the net revenue of the farmers equal to their respective shadow prices. The lack of working capital thus establishes the fact that there is an intense need for expanding credit facilities in the project area.

The basic solution of the LP model indicates the optimum annual net revenues of the owner farmers, owner-cum-tenants and the tenant's worth of Rs. 8541.77, Rs. 7856.16 and Rs. 7309.13 respectively. Obviously differences in the net revenues or economic efficiencies of the farms could relatively be attributed to operational holdings.

### SENSITIVITY ANALYSIS OF THE MODEL

The analysis of parametric change and their effect on linear programming solutions is referred to as "sensitivity" or "post-optimality" analysis. In other words, it is the study of parametric changes and the relative sensitivity of the basic solution to these changes. The most obvious means for analyzing these changes is to record the changes in the original problem formulation, solve again via old. The difference between these two solutions would indicate the effects of parametric changes and their sensitivities.

The results of the basic model indicate that the farmer of a 7.5 acre of farm holding has been unable, at prevailing water supply and capital restriction imposed, to generate the internal surpluses that are required to move systematically towards a higher level of technology, and in turn to enhance economic efficiency of the farm. In the following part of the section, two parameters in the model have been changed to provide a better picture of the interaction between credit, water (technological change) and the growth of output on sample farms in the irrigated area of Khyber Pakhtunkhwa, pk

irrigation water. The activity wise results of the model by tenurial status are presented in the following table 2, in addition, the cropping pattern, the total cropped area, cropping intensity and net revenue are also indicated in the table 2 below:

In this exercise, credit and water are scaled upward in increments of 20 per cent and 25 per cent of the available working capital and water respectively. The consequences of such positive changes in the inputs are presented in the table 3 as follows:

Table 3. Optimal cropped area, cropping intensity and net revenue for a range of credit and water availability

Description	Total cropped area (acre)	Cropping intensity %	Shadow price of credit (Rs.)	Net revenue
<b>1. Owner</b>				
i) A	10.50	140.00	1.90	8541.77
ii) B	10.95	146.00	1.65	9370.15
iii) C	11.15	156.67	1.65	9804.50
<b>2. Owner-cum- Tenants</b>				
i) A	10.30	137.33	1.99	7856.16
ii) B	10.80	144.00	1.65	8241.75
iii) C	11.14	148.53	1.65	8673.15
<b>3. Tenants</b>				
i) A	10.10	134.67	2.01	7309.13
ii) B	10.35	138.	1.65	7727.16
iii) C	11.02	146.93	1.65	8205.26

A= Canal + purchased tube well water with initial working capital of Rs. 3,000/-

S(credit Rs.15000+25% water

B= Increase of credit by 20% + the same amount of 25% water

C = Increase of credit by 20% + 50% water

The results reveal that the net revenue of the owner farmers has increased from, Rs 8541.77 to Rs. 9370.15, when the credit is increased by a rate of 20% of the initial working capital, holding water supply constant. It enhanced to Rs. 9840.50 when water is increased by 25%. The impact of upward shifts in the amounts of credit and water supply can be judged in the same direction on the farms operated by owner-cum-tenants and tenants. The sensitivity of a rise in the quantities of credit and water supply with respect to corresponding increase in the net revenue (economic efficiency) has been remarkable, increasing net revenues by 14.78%, 12.26% and 10.40% on the farms managed by owners, tenants and owner-cum-tenants farms, respectively.

The sensitivity of results indicates that for the addition of credit and irrigated water to the initial working capital and available water, the farmers would still be willing to pay more than the prevailing interest rate and tubewell water charges.

It is evident from the data presented in table 3 that the total cropped area has increased with a corresponding rise in the amounts of credit and irrigation water on all types of the farms. The proportions of increase on the farms operated by the tenants, owner-cum-tenants and the owners have been 9.11%, 8.16% and 6.19% respectively. The results further indicate that the availability of working capital (credit) and irrigation water has been relatively more

sensitive for tenants than for the owners, which reflects that credits have enabled the farmers in general and the tenant operators in particular in purchasing the inputs and in increasing their total cropped area.

The results of the LP model after relaxing the credit and water constraints have shown that the cropping intensities have also significantly increased on all types of the farms.

The data given in Table 3 reveals that relaxing of more water constraints further increases the acreage of the farm crops. Furthermore, the increase in net revenue and rise in the shadow price of credit can contribute towards more credit utilization efficiency.

### CONCLUSION

The above exercises undertaken describe the conditions of the sample farmers, under a variety of assumptions of the availability of Land, Family Labour, Bullock Labour, Capital (Credit) and Water. The results of LP Model suggested that the profit maximizing farmers cultivated not less than 10 acres. The cropping intensities declined due to different tenures from owner farmers to owner-cum-tenants and tenant farmers, which indicate uneven distribution of land holdings and lack of capital in case of tenant farmers. The higher shadow prices of working capital on farms operated by tenants, owner-cum-tenants and owners, clearly indicted the differences of lack of capital on the respective farms. The tenants were at the worst position in this regard.

The Sensitivity Analysis determined that positive changes in credit (availability of capital) and irrigation water might significantly increase the profit (Net Revenue) of the farmers. These results imply that all the sample farmers are utilizing their resources less efficiently than the optimum level of utilization. This might be attributed to the inappropriate agrarian structure.

The possibilities of a trade-off between land and borrowed capital as well as land and water availability, access to credit and tubewell water are clearly substantial for improving the economic efficiency of farms.

The LP model and sensitivity analysis showed that a very small increase in the working capital resulted in a very remarkable rise in the net revenue of the farms. The results of this study suggested that availability of credit would enhance the economic efficiency of the farms. There was a substantial need for an in-depth review of the existing institutional credit structure and major overhaul of credit disbursement procedures for further boosting the farm efficiency.

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